Shaping the Experience of Young and Naïve Probabilists

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Summary

This paper starts by briefly summarizing research which has focused on student misconceptions with respect to understanding probability, before outlining the author's research, which, in contrast, presents what students of age 11-12 years(Grade 7) ***do*** *know and can construct, given access to a carefully designed environment. These students judged randomness according to unpredictability, lack of pattern in results, lack of control over outcomes and fairness. In many respects their judgments were based on similar characteristics to those of experts. However, it was only through interaction with a virtual environment, ChanceMaker, that the students began to express situated meanings for aggregated long-term randomness. That data is then re-analyzed in order to reflect upon the design decisions that shaped the environment itself. Four main design heuristics are identified and elaborated: Testing personal conjectures, Building on student knowledge, Linking purpose and utility, Fusing control and representation. In conclusion, it is conjectured that these heuristics are of wider relevance to teachers and lecturers, who aspire to shape the experience of young and naïve probabilists through their actions as designers of tasks and pedagogical settings.*

Preamble

The aim in this paper is to propose heuristics that might usefully guide design decisions for pedagogues, who might be building software, creating new curriculum approaches, imagining a novel task or writing a lesson plan. (Here I use the term "heuristics" with its standard meaning as a rule-of-thumb, a guiding rule, rather than in the specialized sense that has been adopted by researchers of judgments of chance.) With this aim in mind, I re-analyze data that have previously informed my research on students' meanings for randomness as emergent during activity.

I will begin by deconstructing my title for the paper, starting from the end, in order to expose various assumptions that will help to situate the paper within a particular theoretical stance…

Probabilists: The design heuristics reported below are the result of research on meanings for randomness and chance, aspects of probability which separate the topic from exercises in set theory logic using, for example, Boolean algebra but which are tuned to a modeling perspective on randomness. In fact, I would hope that the designer/teacher with interest in statistics more generally will nevertheless find resonance with the heuristics, which I believe have relevance beyond probability, perhaps even to all areas of mathematics.

Young and naïve: In fact, it seems there are probabilists at one level of naivety or another at all ages, after all, Fischbein's (1975) kindergarten children were reported as being able to discriminate relative frequencies. Furthermore, the much reported studies by Kahneman et al (for example, 1982) and others show how adults, sometimes statistically trained adults, typically make judgments of chance using heuristics, which are subject to systematic bias. Additionally many other misconceptions relating to the understanding of probability amongst a wide range of ages have been identified, for example, the equiprobability bias (Lecoutre 1992) and the outcome approach (Konold 1989). Naivety, or worse fallibility, appears to be endemic. Perceptions of randomness itself have been studied by many researchers, though again the predominant analysis has led to the identification of errors. In reviewing this research, Falk and Konold (1998) identified two categories, generation type tasks, where subjects were required to predict outcomes from a random experiment, and recognition tasks, where people were expected to state which sequences had previously been generated by a random mechanism. Such studies found a tendency for people to anticipate too many short runs of results in a random sequence and to regard sequences containing long runs as non-random. I note that, in Green's studies (1983), which confirmed the tendency of people to reject runs, it is acknowledged that, alongside a failure to understand the role of independency in successive trials, was effective reference to pattern recognition and unpredictability. A constructivist perspective on design requires us to identify such mental resources that might act as building blocks for a more sophisticated understanding. In line with Smith et al (1993), who argue for a re-conceptualization of misconceptions, my intention is to emphasize competence and promote an understanding of how naïve mental resources might be supported as they become increasingly sophisticated, a process diSessa (1993) has referred to as "tuning towards expertise".

Shaping the experience: This phrase is imbued with theoretical assumptions. No self respecting educationalist fails these days to espouse constructivist credentials, forcing us all into admitting that our influence on learning as educationalists is at best indirect. Shaping what students learn is the best to which we can aspire. But the nature, never mind the size, of the influence of factors such as setting, tools, culture, motivation, belief systems, identity and existing knowledge, to name but a few, is not well understood. Nevertheless, I take it as axiomatic that such factors are influential and focus my attention on how designing software tools can have an indirect influence on, in other words *shape*, the student experience.

In this paper then, I shall discuss, at a theoretical level, how designers of software might leverage opportunities available to them within their operational setting to have influence over the way in which learners construct stochastic meaningfulness within that designed world. Although the heuristics, which will be presented, emerge from the design of software to research the relationship with young students' stochastic meanings, the findings will in this paper be re-interpreted to offer conjectures that might resonate for designers in other contexts; here I am thinking of people who might not even think of themselves as designers, such as curriculum planners or innovators and classroom teachers or lecturers. Though I present no new data (previously reported finding are summarized below), I do offer new unpublished findings (though a much longer elaboration exists, Pratt et al, submitted) and a conjectured extension for teachers of statistics at all levels.

The Original Analysis: Existing and Emergent Meanings for Randomness

I provide a brief synopsis here of my research that has previously been widely reported (for example, Pratt 1998; Pratt 2000; Pratt & Noss 2002). This study used a design research approach (Cobb et al 2003) to build a domain of stochastic abstraction, *ChanceMaker*, in which 10-11 year old students were able to simulate everyday random generators (*gadgets)*, such as coins, spinners and dice (Figure 1shows the dice gadget). The students had access to an artificial strength control. By pulling the strength control, the students could simulate the throwing of the gadget though in fact the strength would have no effect on the actual result, only on how long the simulated animation would last. Many of the gadgets were by default programmed to behave in non-standard ways, perhaps with a bias to one outcome or another. Thus, in Figure 1, the die is biased towards the outcome 6. The students were challenged to identify which gadgets were "working properly" and which were not. They were then further challenged to "mend" any broken gadgets so that they behaved according to the students' expectations of randomness. The students could control the behavior of the gadgets by editing the workings box; a student encountering the working box in Figure 1 might decide to remove one 6 so that it would then read "choose-from [1 2 3 4 5 6 6]", and later after further amendment to read "choose-from [1 2 3 4 5 6]". The students were also able to generate as many results as they wished in a relatively short space of time using a repeat control. In Figure 1, the student has generated 20 throws and the corresponding pie chart.

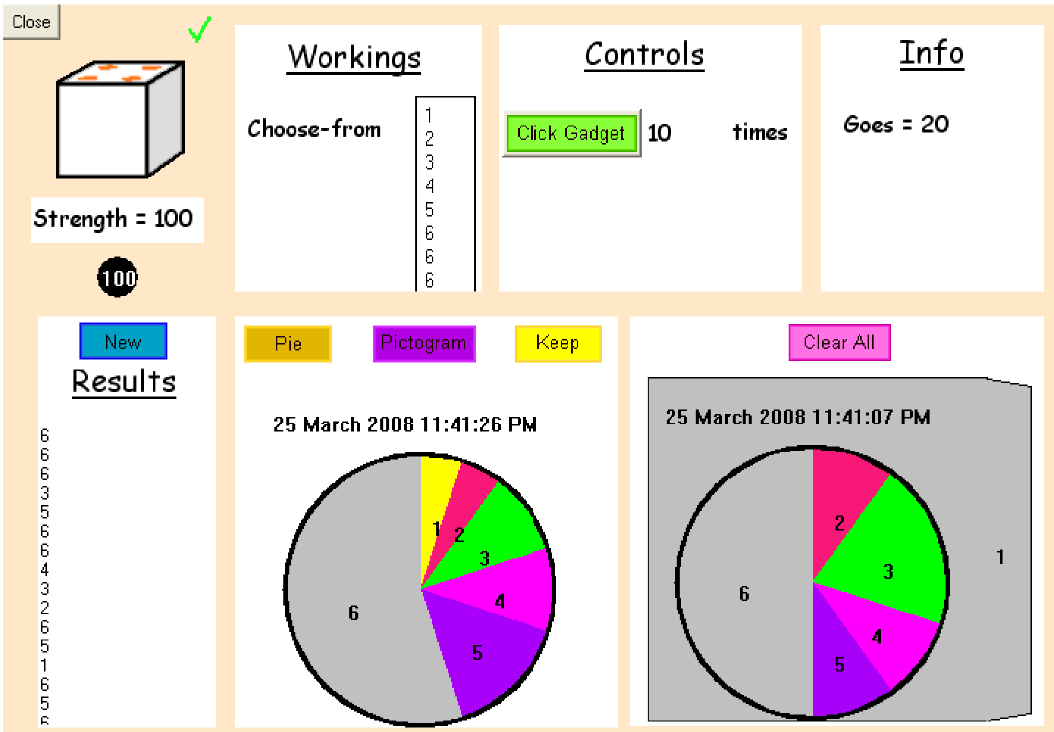

Figure 1: The ChanceMaker dice gadget can be opened up to reveal a "broken" workings box. Here the student has created a pie chart from 10 throws, kept that picture (on the right), and then created a second pie chart from 20 throws (on the left).

The window, provided by the iterative design process, on the students' emergent thinking about randomness (Noss & Hoyles 1996), indicated that the students used a range of expert-like meanings described as *local* (in the sense that these were all situated in the short-term here-and-now); the students would recognize as random those gadgets which were unpredictable, uncontrollable, unpatterned or fair, a result consistent with Toohey's findings (1996). Fairness was an important resource that the students were able to use to construct more sophisticated *global* meanings (an aggregated long term perspective on randomness).

In their interactions with ChanceMaker, the students began to articulate heuristics which were situated but nevertheless appeared to describe behavior across the gadgets. These articulations could be characterized, for example, as "the more times you throw the dice, the more even is the pie chart" to describe the observation that fairness in the pie chart (as opposed to in the appearance of the gadget) would manifest itself as equal size sectors when trials were repeated large numbers of times. This causal-like heuristic (more trials cause a more even pie chart) seemed to capture the essence of what the expert might refer to as the *Law of Large Numbers*. Noting however that this heuristic seemed to fail when the number of trials was small, or when the workings box was not fair, some students began to articulate a second heuristic: "The more even is the workings box, the more even is the pie chart, provided the number of trials is large", which appears in expert terms to acknowledge the role of distribution.

A Re-analysis: Heuristics for Software Designers who Would Like to Shape the Experience of Young Probabilists

More recently, I have carried out a re-analysis of the data in my original research from the perspective of the software design. Whereas, in Section 2, the focus was on the students' thinking-in-change about randomness within the ChanceMaker setting, in this section, I reverse the emphasis to reflect on the ChanceMaker design in the context of how students' thinking changed. This re-analysis has yielded four main design heuristics that were significant in shaping the student ChanceMaker experience and may have wider relevance: Testing personal conjectures, Building on current knowledge, Linking purpose and utility, Fusing control and representation.

Testing personal conjectures

The ChanceMaker students articulated the four local meanings for randomness described above but the students' use of these meanings was very sensitive to small (from the designer's point of view) changes in the setting. Thus, a student might characterize a spinner as random because of the feeling of being unable to control the outcome (lack of control), perceived through the spinning action, but the same student might judge a dice to be random even without throwing it by reference to its apparent symmetry (fairness). Some situations might have been seen as exhibiting more than one of the behaviors in contradictory ways. For example, an irregular spinner, which was not fair, might have been seen as not random fro that perspective and yet also random since it could not be precisely controlled. Such contradictions were unproblematic for the student, who simply used whichever meaning was stimulated by surface aspects of the situation. In this sense, the local meanings for randomness appeared to exist without reference or connection to each other. diSessa (1993) has proposed that such observations are consistent with a view of conceptual change as fragmented, knowledge being conceived of as distributed across many small pieces. This *knowledge-in-pieces* profile of the students' local meanings for randomness led to the careful incorporation in ChanceMaker of opportunities to test out personal conjectures, since it was felt that students needed to be able to recognize that their meanings lacked some robustness if they were to construct more sophisticated meanings in the process of tuning towards expertise. As a result, ChanceMaker was designed to incorporate:

- Animations and histories of results, intended to suggest that patterns in sequences of data generated by a gadget as identified by the student were illusory in the sense that on further experimentation the patterns were not sustained.
- An editable workings box, so that students could test whether for example the order of the numbers on the spinner was important.
- A control over the strength of the throw, which was in fact a *redundant control* in the sense that strength had no effect on outcome, intended to suggest that strength was less important than might have been assumed.

Building on current knowledge

Testing personal conjectures in a well-designed environment can lead to a position of cognitive conflict (see, for example, Piaget, 1985). Piaget explains how all development emerges out of attempts to resolve cognitive conflict in a process of equilibration. While, according to Piaget, assimilation requires adaptation by simple integration of the conflicting ideas into an existing mental structure, breakthrough development involves a restructuring referred to as accommodation. However, as a designer, it is necessary to provide pathways that both anticipate, indeed stimulate, such cognitive conflict and yet at the same time those pathways need to offer a resolution of the imbalance. In the preamble, I described my dissatisfaction with simply reporting on students' fallibility and indeed I have reported above on students' expert-like local meanings for randomness, and the root of this dissatisfaction for me as a designer lies in the failure of misconceptions research to elaborate such pathways. I sought to design into ChanceMaker a mechanism by which these local meanings could be exploited and coordinated, a process motivated by the notion of tuning towards expertise. In this sense, I go beyond the often espoused notion of cognitive conflict by aiming to provide a pathway through that conflict to some emerging resolution. With this aspiration in mind, I built into ChanceMaker:

- Gadgets, which were intentionally designed to look and behave as far as possible like their everyday counterparts, so that prior intuitive knowledge might be triggered. At the same time, the gadgets could be opened up and (a)mended, an action that is usually very difficult, if not impossible, with everyday materials. It is often commented in critiques that perhaps the students did not regard the computer's quasi-randomness as the same phenomenon as might be observed through an ordinary material die. In fact the data in the study does not support the idea that the students were concerned about such a distinction. The students were occasionally skeptical in the first instance but such doubts were forgotten or resolved when they experimented and found that the computer gadget obeyed the students' own criteria for judging randomness, namely the four local meanings for randomness.

- Graphical representations, such as the pie chart, which foregrounded fairness, not as a property (or non-property) of the dice itself but of the aggregated results it generated, opening up the possibility that the students would re-attach their notion of fairness from the gadget to its outcomes, providing the necessary pathway to a resolution of cognitive conflict.

Linking purpose and utility

Elsewhere, I have discussed the necessity that tasks are designed in ways that students regard as purposeful (Ainley, Pratt & Hansen 2006). Purpose is seen as a design construct in that those who might aspire to shape student experience might attempt to design settings and tasks with engagement of students in mind. My experience has suggested that design efforts can be steered by certain heuristics such as building and mending tasks, problems that raise curiosity, and areas of controversy. At the same time, I have argued that purpose in itself is insufficient and must be linked to utility, a cognitive sense of the scope of the key mathematical concepts. For example, a purposeful task may or may not lead to a sense of the domain of applicability of a situated meaning for a key concept, in other words its level of specificity or generality. Designing to link purpose and utility emerges from constructionist aspirations (Harel & Papert 1991) to involve students in their own abstracting process.

In ChanceMaker, the students were challenged to mend broken gadgets and this proved to be an engaging and consuming pursuit for the students. It was however key that this task also led to a sense of utility for distribution as represented by the workings box. The students came to understand how the distribution, or rather the workings box, is a predictor of the proportions of results, but that the validity of this predictor is somehow predicated on the number of trials. From a design perspective, the crucial decision was to point the ChanceMaker activity around mending which led inevitably to engagement with the workings box.

Fusing control and representation

In fact, that utility stemmed from using the workings box to control how the gadget behaved. The constant editing and re-using of the workings box, coupled with feedback in the gadget's animation, the history of results and graphs of aggregated results, rendered the controlling workings box meaningful. Indeed, gradually as the students became increasingly familiar with the nature of that control, they were able to predict behavior of the gadget from the appearance of the workings box. In this sense, the workings box had become a representation for them of distribution. It seems that the link between purpose and utility can be facilitated by building salient mathematical representations as controls within the domain of abstraction in question.

Conjectures on How Putative Designers Might Shape the Experience of Naive Probabilists

The four design heuristics above have emerged from the systematic study of students' thinking-in-change about randomness. To what extent are those heuristics tied to the specific operational setting, involving software design, young students and randomness? This is, of course, difficult to judge without further research. However, there are some pointers which enable me to assess whether these following conjectures merit further investigation: (i) Meaningfulness of probabilistic concepts might be enhanced by providing space for the testing of conjectures; (ii) Meaningfulness of probabilistic concepts might be enhanced by identifying and building on current student knowledge; (iii) Meaningfulness of probabilistic concepts might be enhanced by linking purpose and utility; (iv) Purpose and utility of probabilistic concepts might be linked by fusing control and representation.

Conjecture 1: "Meaningfulness of probabilistic concepts might be enhanced by providing space for the testing of conjectures"

In Higher Education there are logistical constraints such as the very high student to teacher ratio (at least in the conventional lecture format) that push the pedagogic approach towards a stand-and-deliver format. Lectures proceed at the speed of the lecturer, which is unlikely to provide the time and conditions under which students would be well-positioned to test personal conjectures. Nevertheless, such students are relatively independent and may have out-of-lecture space for testing their own ideas, if only they knew how to go about this task. In secondary schools (at least in the UK), the assessment regime pressurizes teachers and students into covering the syllabus, which can militate against opportunities for sense-making by students. Nevertheless, the need to provide this sort of space does not appear to be fundamentally a consequence of using digital technology and is consistent with reform agendas in many countries. ChanceMaker was designed to provide feedback in numerous forms and this feedback was crucial in helping the students to recognize whether their personal conjectures held explanatory power. Feedback forms are highly dependent on the structuring resources available in any particular setting and may be difficult to organize in traditional contexts.

Conjecture 2: "Meaningfulness of probabilistic concepts might be enhanced by identifying and building on current student knowledge"

Research on mathematical cognition in general has traditionally placed its emphasis on identifying student misconceptions. Though such work persists, it is now more common to consider the cognitive and socio-cultural conditions under which normalized thinking might be constructed or supported. In statistics education, it is still the case that much research focuses on people's fallibility. A new research effort is needed to understand the intuitive strata that underpins student thinking and new methodologies have to be employed in order to exploit that understanding in the way we offer up our subject disciplines. I note that the ChanceMaker study not only identified the key role, for example, of students' appreciation of fairness but also found how students could make use of that naïve knowledge, in particular the students' notion of fairness, to construct a more sophisticated understanding of randomness and distribution.

Conjecture 3: "Meaningfulness of probabilistic concepts might be enhanced by linking purpose and utility"

It is common to hear students both in Higher and Secondary Education to despair of the lack of connection and relevance of mathematics and statistics to their lives. However, *purpose* is no more of a synonym for *relevance* than is *utility* for *usefulness*. The ChanceMaker students were not conducting an experiment that they could see would be useful for them outside of school or indeed in their future work. Neither did it have obvious and direct value for forthcoming examinations. Rather, the purpose was generated because their curiosity was peeked. They needed to know, for their own personal satisfaction, how the gadgets might be mended because of some sort of innate human response to problems of this kind. The tools provided operationalized that need so that the students could act concretely and creatively towards satisfying that need. They were in fact prepared to suspend any sense of reality while they engaged with the task. The utility that was constructed was related to an appreciation of how the workings box impacted upon the pie charts in the short term and in the long term. In this sense, they learned a fundamental truth about the scope of distribution, though for them the knowledge was far too situated to be appreciated in such grand terms. Again, linking purpose and utility may be easier when using digital technology (though in my experience it is still difficult) but utility is a fundamental, though often ignored, aspect of conceptual understanding, and must, in my view, be given a much higher priority by teachers and curriculum developers.

Conjecture 4: "Purpose and utility of probabilistic concepts might be linked by fusing control and representation"

I see no reason why in principle the above three conjectures should not have validity in conventional settings related to probabilistic learning, or indeed more generally mathematical learning (even if the logistical constraints make them difficult to apply). Conjecture 4 may however remain an obstacle even in principle for conventional settings. Papert (1996) has argued that in conventional settings, it is normal for mathematics to be learned in a way which is somewhat alien. Conventionally, mathematical concepts need to be defined and computed before any rich sense of the concept is constructed. For example, students painstakingly learn, year after year, how to draw each and every type of graph and often fail to appreciate interpretation and other uses for graphs. Students learn how to calculate mean, mode and median and their separate definitions without necessarily understanding how the notion of average might be constructively employed towards a specific aim.

During inception, mathematical conceptual knowledge tends to live in a world disconnected from life in general and indeed from other mathematical concepts. Only much later might those connections begin to emerge and might the scope and relevance of the concepts take on meaning. Harel and Papert argue that this presentation of mathematics is an inversion of what happens when most non-mathematical ideas are encountered. In life outside of mathematics, people tend to learn through using. According to their *Power Principle,* technology provides a means of inverting that inversion by *phenomenalising* (Pratt 1998) mathematical ideas into quasi-concrete objects that can be manipulated on-screen as if they were everyday phenomena. In these circumstances, it is possible to use mathematical concepts (or at least on-screen representations of them) without having already struggled with their definitions and computational methods. For example, pedagogic techniques can exploit the graphing capabilities of spreadsheets so that emphasis is removed from computation and low level skills and increased with respect to high level skills such as interpretation. A particular case can be found in the use of *active graphing* where a pedagogic technique encourages the students to construct an analytical utility for graphs, in addition to the conventional presentational utility (Ainley et al. 2001).

Conjecture 4 is fundamentally tied up with the Power Principle. Epistemological analysis of a learning domain can identify likely key mathematical or statistical concepts. Psycho-pedagogical analysis can identify unconventional representations which connect with the intuitive strata of existing knowledge amongst the target student population and provide them as controls within the virtual domain of abstraction. Through engagement with those controls in a purposeful task, the students can begin to construct the control as a representation. So, in the case of ChanceMaker, the concept of distribution was phenomenalised to become the workings box, which first acted as a control over the behavior of the gadget but later also became a representation of proportions of outcomes, in much the same way as experts would think about distribution. Harel and Papert have shown us how digital technology is especially well-suited for the task of phenomenalising mathematical representations so that they can be used as controls. It is not so clear that this trick can be easily performed in more conventional settings.

In conclusion then, in this short paper, I have attempted to summaries one instance of how reflection on software design can lead to the identification of design heuristics. In this case, four such heuristics were identified. The scope of those heuristics to apply to contexts involving (i) older students, (ii) statistical or even mathematical learning, or (iii) teaching without necessarily the use of technology are not yet well understood. However, I have discussed the opportunities and difficulties that might be encountered in extending the heuristics to designers in other operational settings, such as teachers in classrooms.

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