Hands-on Modelling with Wason Cards and Tinker Cubes: First Steps in Logical and Bayesian Reasoning in Fourth Grade

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Summary

Wason cards were originally designed for an experiment evaluating subjects' reasoning abilities. Our claim is that they may be used as an educational tool in the classroom and their didactical value is enormous. Fourth graders can be trained in logical reasoning by playing not only with four Wason cards, as in the original experiment, but with samples of them and with various contexts, e.g., with sentences about daily life and with colours instead of symbols. The transition from logical to probabilistic reasoning can be made palpable to fourth graders through ritualized games with Wason cards. Yet Wason cards have limitations, which become apparent when several features are jointly to be taken into account. For these cases, tinker cubes and tinker towers can be, as we show subsequently, adequate tools for an enactive treatment of proportional and probabilistic reasoning in fourth grade.

Introduction

The time interval during which enactive modelling of mathematical situations can be implemented in school, like by counting marbles and assembling cubes, is becoming shorter because of the early inception of computer activities. Yet it is an old wisdom confirmed by research on developmental psychology (Hirsh-Pasek & Golinkoff, 2003) that playing with simple traditional toys and playing with each other is not just healthy but can be successfully incorporated as classroom activity to enhance learning. Elementary arithmetic and elementary geometry have traditionally profited from playful activities in the classroom. Statistics and probabilities offer even greater possibilities for "learning through play", because the essence of these disciplines is sampling, sorting, betting, and gambling. The attitude of playful discovery is, so to speak, an intrinsic characteristic of stochastic thinking.

That children have probabilistic intuitions is an empirically tested thesis widely illustrated in the pertinent literature. It is beyond the scope of this paper to present an overview of this literature. We base our convictions on the work of Fischbein and the large community that confirmed and specialised his discoveries. At the same time, it is by now an established attitude to avoid using terms like "probability" in classrooms with young children. The current position, at least in Germany, is that *some* aspects of probability are part of children's reasoning but not all. There are experiments that prove how good fourth graders are at comparing events as to their likelihood, although they make mistakes assessing the probability of the union of events. Typically when they estimate − in percentages − the chances that the different teams win, say, the Champions League Cup, the order relations among their estimates are adequate although these estimates for different teams often add up to much more than 100%. Children have intuitions and can be trained in judgments on whether a specific event is "more likely" than another, or whether an event is "very likely", "completely sure" or rather "unlikely" (Neubert, 2007; Martignon & Krauss, 2007). They also enjoy discovering that coins or dice may be "unfair". First graders are able to distinguish "unfair" from "fair" dice and have good intuitions of how an "unfair" die could be constructed (Martignon & Krauss, 2007).

Our contribution is devoted to a different set of tasks than those typically treated in the literature. We describe playful yet ritualized activities for fourth graders that have been conceived for training children in the subtleties of logical implication and then extending strict implications to conditional probabilities. The implication of the kind "If *P* then *Q*" can be extended and transformed into a probabilistic statement of the type "the conditional probability of *Q* given *P* is high". These two statements are, of course, not equivalent even the probability of *Q* given *P* is 1. We will demonstrate how children may learn to distinguish between deterministic implication and probabilistic conditioning. Cards and tinker cubes will serve to model these situations.

Playing with Wason cards

A fundamental contribution to the understanding of how humans perform logical implications was provided by a series of experiments initiated by Wason in the late sixties. Wason tested subjects by showing them 4 cards on a table. The cards had letters on one side and numbers on the other. The subjects saw the following configuration:

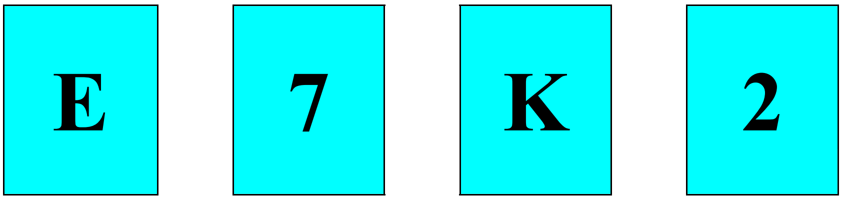

Fig. 1: The original four cards in the Wason selection task.

The subjects were confronted with the following question: *Which cards do you necessarily have to turn around in order to check, whether the following rule holds for the set of 4 cards?*

> "If one side of a card exhibits a vowel, its other side must exhibit an odd number"

The correct answer is "E and 2", because there is one card, which is forbidden by the rule, namely the card with a vowel on one side and an even number on the other. Most subjects (more than 85%) gave a "wrong" answer, either pointing at cards E and 7, or at card E alone. Only 13% of all subjects noticed that card 2 has to be turned around, because if the reverse side exhibited a vowel, the rule would not hold. Wason's results seemed to point to human flaws in logical reasoning.

This conclusion was contended by later experiments carried out by Johnson-Laird, Legrenzi & Legrenzi (1972) in which the abstract context of the binary features "vowels ↔ consonants" and "even numbers ↔ odd numbers" was replaced by a context typical of social contract situations. Let us look at a famous example of this type: let each card represent an envelope to be sent by mail, with its destination written on one side and the value of its stamp on the other.

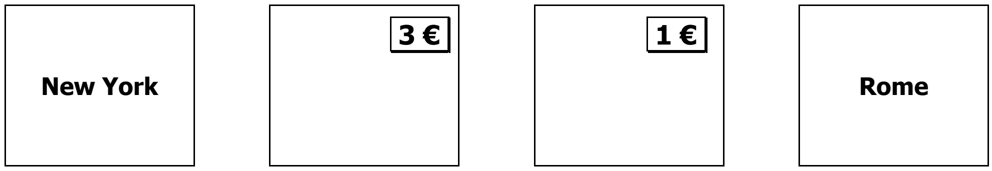

Fig. 2: Four cards for an if-then rule from the "postal" contract.

Assume the rule is:

"If the letter goes to America then it requires at least 2.50€"

The surprising result for this type of rule was that with tasks like this one, stemming from social contracts (the stamps required are established by postal decisions implicitly accepted by consumers), more than 85% solved the task correctly. A flurry of other experiments with rules from the social contract context motivated the theory of the *cheating detection module* on the one side and a historical approach to the evolutionary development of logic on the other side, as the normative system emerging from a "cognitive module" for cheating detection (e.g., Cosmides, 1989; Cosmides and Tooby, 1992). Our flaws with implications in abstract settings, Cosmides and Tooby claim, are a mere sign of non-adaptation (like being unable to recognize colours under an unusual artificial light).

In our studies we adapted the Wason task to a classroom setting and evaluated its educational potential; we transformed the task by modifying not just the context of the four cards, but also the number of the used cards. Switching from 4 to 32 cards, for instance, modified the character of the design. We realized that children performed well in everyday situations not necessarily cast in social contract phrasings (see Figure 5).

In 6 fourth classes in and near Stuttgart we let children sit around tables in groups of four and organized activities for 45 to 90 minutes (with a break of 10 minutes) with Wason cards. The four children at each table were subdivided into two teams. The Wason cards we used included the original set with letters and numbers, sets with other symbols and/or short statements ("If you help me in my homework you can borrow my bicycle"), and sets of coloured cards with no symbol on them. In the typical situation of our studies the children manipulate the cards in their hands before the game starts. Thus children can see both sides of the cards. In what follows we describe one of the empirical studies we performed.

1. We used two pairs of colours that children usually see as binary pairs,
   namely "red ↔ blue" and "black ↔ white".
   Our Wason cards were either red or blue on one side and either black or white on the other.
2. The instructor wrote a "rule" on the blackboard:

"If one side of a card is red then the other side must be white"

We had two modalities:

Experiment I. For three of the classes, only four cards were placed on the tables in an array like the following:

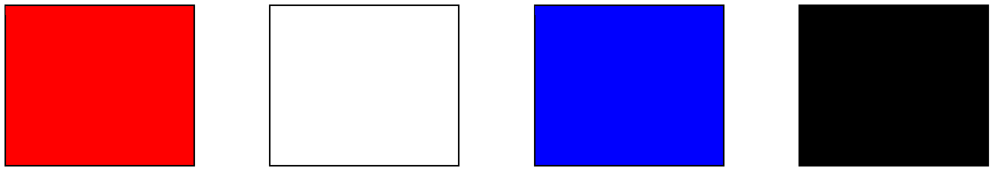

Fig. 3: Four cards with blue/red on one side and white/black on the other side.

Experiment II. For the other three classes, 32 cards (8 red-white, 8 red-black, 8 blue-white, 8 blue-black) were taken out of a little red bag containing large amounts of cards and placed randomly on each table, with the only constraint that each colour was on top at least four times). The instructors wrote the following question on the blackboard, followed by the game instructions, which were read (and explained) loudly:

> "Which cards should be turned around to check whether the rule is true?
>
> At your turn you have to either point to such a card – without turning it around - or to make the statement that no further step is necessary and the game is over. The instructor, or an assistant, also sits at the table and watches the teams playing. At each turn he/she judges whether the indicated card should indeed be turned around. In case of a mistake the team that committed the mistake is eliminated and the other team wins. In case of no errors the team that ends the game wins. Which team starts the game is decided by tossing a coin.
>
> The instructor (or the assistant) waits until the game is over in order to turn the cards around."

There were remarkable differences between the two experimental situations I and II. In Experiment I most teams that played the first move (34 out of 36) began by pointing at a red card. In the next move, less than half (12 out 36) of the teams suggested to turn around the black card. The rest either pointed at the white card or stated that the game was over.

In Experiment II we observed that children were more far more at ease. The game appeared to be more natural than the one proposed in Experiment I, and only in very few cases (10 out of 72), a team eventually indicated a "wrong" card and lost the game. The discussions at the tables after the game were partially reported by the assistants. All children had realized that red-black cards were "forbidden". Children were tested individually three weeks after the intervention with sets of cards with other contexts, e.g., the original Wason cards with letters on one side and numbers on the other.

Fig. 4: Playing with 32 Wason cards.

Only less than 20% of the children performed poorly. We attribute both the ease of children in Experiment II and their good performance to the different functions of the cards set. In Experiment I cards are abstract representatives of an artificial situation (4 cards for the 4 cases “*P* and *Q*”, “*P* and not *Q*”, “not *P* and *Q*”, “not *P* and not *Q*”). Also the situation of real games and competitions provided by the setting of Experiment II motivated a greater concentration. In fact, Experiment II was perceived as a real game, while Experiment I was experienced as kind of strange test. The differences in attitude in the two settings were indeed surprising. We have only fragmentary and speculative explanations: In Experiment II children are working with a sample and, as we believe, become “intuitive statisticians”. This hypothesis has to be tested by means of further investigations. It is important to note that children did not necessarily tend to uncover all “modus ponens” cases first and then pass to the “modus tollens” cases.

From Certainty to Uncertainty

Fischbein, among others, pointed out that children tend to grow with the belief that *“ambiguity and uncertainty are not acceptable in scientific reasoning”*. Scientific reasoning, since Aristotle, has been ruled by what is called classical logic until the Enlightenment when scientists of the calibre of Pascal and Laplace enhanced logical inference into a new instrument, capable of dealing with uncertainty, namely the probabilistic calculus (Daston, 1988). One of the recommendations of the NCTM standards and the adaptations thereof in Germany is that children should early be trained in “good” reasoning, not just in the mathematical context. Good reasoning comprises not just practicing deduction but also performing inductive conclusions.

In this contribution we want to stress both the importance of logical reasoning and the importance of a conscious transition from certainty to uncertainty. Following a “historic” trajectory we let children experience the transition − from logic to probability − in our exploratory studies. The children had to realize by themselves that “if-then sentences” have limitations, that they may fail to provide adequate descriptions of real situations and that a “looser” form of conditioning becomes necessary for describing real life situations. In two subsequent 2 hour blocks children experienced the transition from Wason cards to tinker cubes for representing individuals. Children were again organized in teams of two and sat at tables. We used the following introductory statement:

> “If you are a boy then you like computer games”.

We provided plain cards to all children, so that they could produce the Wason cards for this rule. The aim was to examine Wason cards for two features like the following:

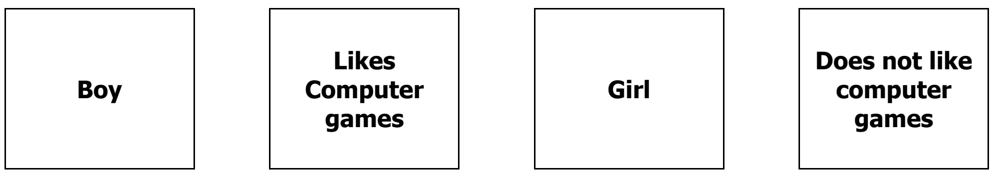

Fig.5: Wason cards with if-then rules from children’s preferences.

Questions written on the blackboard were designed to motivate the use of cards for representing individuals. The questions were:

1. Is it true **in this class** that if you are a boy you like computer games?
2. How is it with girls?
3. How do we find out?

The first step was to introduce lots of "empty" (unwritten) cards. Cards now became representatives of individuals, colours being used to encode features. Children had to colour the cards: Each child in the classroom was now represented by one card; *blue* encoded "boy" and *red* encoded "girl"; *white* encoded "likes computer games", *black* encoded "does not like computer games". The two sides of each card had to be coloured accordingly: a boy who liked computer games was represented by a *blue-white* card, while a *red-black* card represented a girl who did not like computer games. Children were asked to state whether they liked computer games and colour their card accordingly. A first set for the class was placed on the teacher's desk and copies of it were placed on the individual desks by the children. Once the class was represented by a set of cards on each table it became easy to count and calculate statistics. Contingency tables were introduced as well as simple ways to report the data.

Tinker Cubes

Wason cards are symmetric and they can only encode two binary features. Because one of their sides is always hidden they provide a mystery situation that converts children into "detectives". We saw a great advantage in moving from the Wason cards to materials that eliminate mystery. Tinker cubes is the name we gave (Martignon & Kurz-Milcke, 2006) to small cubes made of plastic in different colours, that can be assembled to form tinker towers:

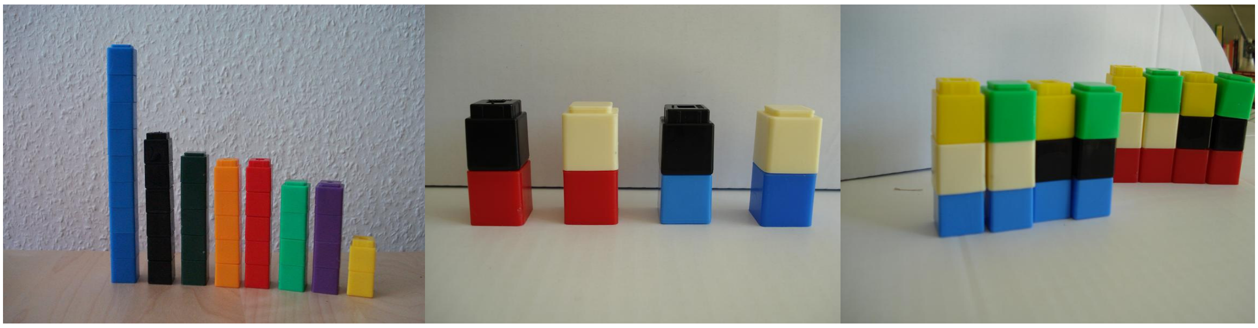

Fig. 6: Tinker towers representing
a. histograms. b. conjunctions of 2 features. c. conjunctions of 3 features.

These materials have been sold as educational tools for more than ten years; our innovation consists in making a new use of these rather old materials. Tinker towers, obtained by assembling together two or more tinker cubes, eliminate the mystery-element of Wason cards, because all features are visible at the same time (without reverting cards). Furthermore, tinker towers can be seen as ordered "vectors" of feature values, represented by colours. Children in all our six classes (the same mentioned in the previous sections) were instructed in an enactive construction of statistical situations with the help of tinker-towers. One activity was the construction of their "fruit preferences" (in Figure 6a the blue tinker tower represents all children of classes 1a and 1b in the Altenburg School in Stuttgart who prefer banana over other fruits, the red tower represents children preferring strawberries).

In one activity they had to encode *boys* as represented by *blue* cubes and *girls* by *red* cubes. *Liking computer games* was encoded by the colour *white* and *not liking computer games* was encoded by the colour *black*, in analogy to the encoding with the coloured Wason cards. But now a new feature was added: *having or not having Math as one's favourite subject*. The pair of colours *yellow-green* was introduced to encode the values of this new binary feature (Figure 6c). The next step was to perform quantified categorizations and introduce the concept of *proportions*. The instructor asked, for instance, what proportion of girls likes computer games? What proportion of those who like computer games are boys?

At the end of the two blocks a written test was performed. Children were given sheets with data on two fourth classes in the United States. Here they had to deal with two features or cues:

1. Liking or not liking computer games
2. Liking or not liking Math as one's favourite subject

Constructing, sorting, counting and assessing proportions of tinker towers in *nested* sets of feature conjunctions is an activity that children perform with ease and pleasure and a first step towards to a fruitful combination of descriptive statistics on one hand and "natural" Bayesian reasoning on the other. Most children (≈70%) in each of the six classes were successful in dealing with all kinds of conditional assessments and could answer questions like "Do boys who like Math necessarily like computer games?", or "Are those who do not like computer games and yet prefer Math over other subjects more likely to be girls?" We note, in passing, that there is good empirical evidence that children are "natural Bayesians"(Kurz-Milcke & Martignon 2006; Zhu & Gigerenzer 2006).

From proportional to probabilistic thinking

Proportions of tinker towers may be used as enactive representations of proportions of subpopulations of individuals - with certain feature conjunctions - out of larger populations. These proportions are a first step towards probabilistic thinking. The use of a *plastic urn* as a container for populations of tinker towers that can be held and "shaken" by one child while another child draws "blindly" from it, adds a probabilistic aspect to tinker towers. In our interventions urns were used consistently and with two functions: on the one hand they are static containers of tinker towers that can be counted (descriptive statistics), on the other hand they can be "shaken" to mix up the towers to allow for random drawing. After constructing histograms made of tinker cubes as in Figure 6a, or populations of tinker towers as in Figures 6b, and 6c, children had to place these towers into urns. Then they proceeded to shake the urns and draw from them, while "betting" on the outcomes. Several experiments evaluating these activities are presently carried out.

Conclusion

In explorative studies we have investigated the potential of Wason cards both as didactical tools for logical reasoning and for a smooth transition from strict implications to conditional statements. We have described the consistent use of tinker cubes and tinker towers for enhancing elementary proportional thinking and first steps in probabilistic comparisons. Our first analyses encourage further experiments and evaluations.

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